

Measures of Central Tendency and Dispersion Notes

Measures of Central Tendency

Measures of central location pinpoint to us central values or commonly called averages. There are three measures of central tendency these are mean, median and mode.

Measures of Central Tendency for Ungroup Data

Mean

To find arithmetic mean, add all the items of observations, then, divide the sum by the total number of observations. In symbols,

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N} \text{ Where } x_i = \text{ith observation, } N = \text{total number of observations and } i = 1, \dots, N.$$

Median

The median is the midpoint of an array of numbers or observations. Let us denote the \tilde{x} .

If a set of data contains an odd number of observations, the median is only the middle observation that divides the set into two equal parts.

Mode

The mode is the observation that appears frequent number of times in a distribution.

Sample Problem 1: Solve problems involving mean.

1. The grades of student B in five subjects are 78, 88, 89, 90, and 95. What is her mean grade?

Solution:

$$\bar{x} = \frac{78 + 88 + 89 + 90 + 95}{5} = \frac{440}{5} = 88$$

2. The distribution of the student's Scores in a Statistics Examination. Find the mean score of the students.

x	Frequency
10	7
11	12
20	20
24	12
33	5
Total	56

Solution:

$$\bar{x} = \frac{7(10) + 12(11) + 20(20) + 12(24) + 5(33)}{56} = \frac{1055}{56} = 18.84$$

Measures of Central Tendency and Dispersion Notes

Sample Problem 2: Solve problems involving median.

3. Find the median score of sophomore students in Chemistry quiz.

12, 34, 23, 14, 16, 33, 41, 35, 10, 45, 25, 24, 50

Solution:

Write the observations in ascending order.

10, 12, 14, 16, 23, 24, 25, 33, 34, 35, 41, 45, 50

Thus, the median score is 25

4. The ages of the patients at the pediatric ward of hospital X are 10, 2, 5, 6, 5, 8, 9, and 9. Find the median of the patient's ages.

Solution:

Write the ages in ascending order.

2, 5, 5, 6, 8, 9, 9, 10

$$\tilde{x} = \frac{6 + 8}{2} = \frac{14}{2} = 7$$

Sample Problem 3: Solve problems involving mode.

5. What is the mode of the students' scores in a Statistics test?

The scores are as follows: 12, 13, 12, 11, 10, 20, 24, 25, 10, 22, 20, 13, 16, 18, 20, 20, 20, 20.

Solution:

Score	Frequency
10	2
11	1
12	2
13	2
16	1
18	1
20	6
22	1
24	1
25	1

highest frequency

Therefore, the mode is 20 since it is the observation or score that appeared the most number of times.

Measures of Central Tendency and Dispersion Notes

Measures of Dispersion

The measure of dispersion will enable you to know how varied the observations are, whether there are extreme values in the distribution, or whether the values are very close to each other. If the measure of dispersion is zero, it means that there is no variation at all and that the dispersion are all alike, or homogeneous. Otherwise, they are heterogeneous.

The common measures of dispersion are range, variance and standard deviation.

Measures of Dispersion for Ungroup Data

Range

The *range* is the simplest form of measuring the variation of a distribution. To get the range, subtract the lowest score or observation from the highest score.

$$R = \text{Highest Observation} - \text{Lowest Observation}$$

Variance

Variance is another measure of dispersion, which can be used instead of range. The variance considers the deviation of each observation from the mean. To obtain the variance of a distribution, first, square the deviation from the mean of each raw score and add them together. Then divide the resulting sum by N or the total number of cases.

A. Population Variance

$$\sigma_N^2 = \frac{\sum (x - \mu)^2}{N}$$

Where σ_N^2 = population variance, x = raw score, μ = population mean and N = number of observations

B. Sample Variance

$$s_N^2 = \frac{\sum (x - \bar{x})^2}{N - 1}$$

Where s_N^2 = population variance, x = raw score, \bar{x} = population mean and N = number of observations

Standard Deviation

The *standard deviation*, σ for a population or s for a sample, is the square root of the value of the variance. In symbols,

A. Population standard deviation (σ)

$$\sigma = \sqrt{\sigma_N^2}$$

B. Sample Standard Deviation (s)

$$s = \sqrt{s_N^2}$$

Measures of Central Tendency and Dispersion Notes

Sample Problem 3: Solve problems involving measures of dispersion.

6. Find the range of the scores of sophomore students in Chemistry quiz.

12, 34, 23, 14, 16, 33, 41, 35, 10, 45, 25, 24, 50

Solution:

$$\text{Range} = 50 - 10 = 40$$

7. Find the population and sample variance and standard deviation of the following distribution:

34, 35, 45, 56, 32, 25, and 40.

Solution:

Variance:

$$\bar{x} = \frac{34 + 35 + 45 + 56 + 32 + 25 + 40}{7} = \frac{267}{7} = 38.14$$

x	$ x - \bar{x} $	$(x - \bar{x})^2$
34	4.14	17.14
35	3.14	9.86
45	6.86	47.06
56	17.86	318.98
32	6.14	37.70
25	13.14	172.66
40	1.86	3.46
267	53.14	606.86

Population Variance

$$\sigma_N^2 = \frac{\sum(x - \mu)^2}{N} = \frac{606.86}{7} = 86.7$$

Sample Variance

$$s_N^2 = \frac{\sum(x - \bar{x})^2}{N - 1} = \frac{606.86}{6} = 101.14$$

Population Standard Deviation

$$\sigma = \sqrt{\sigma_N^2} = \sqrt{86.71} = 9.3$$

Sample Standard Deviation

$$s = \sqrt{s_N^2} = \sqrt{101.14} = 10.05$$